Negation

$\neg$ = not

$\neg((p \wedge q) \rightarrow r)$

rules

$\neg(\neg p) \equiv p$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Predicate logic

P(x) means x is purple

x = grape → P(grape) → grape is purple

P(x) is true for some fruits x

P(x) is true for all fruits x

Quantifiers!

$\exists$ (there exists) $x \in$ fruits such that P(x)

$\forall$ (for all) $x \in$ fruits, P(x)

$\underbrace{\exists x, y}_{\text{bound variables}} \in$ fruits $\underbrace{\text{such that } (P(x) \wedge \neg P(y))}_{\text{scope}}$

$Q(x) =$ x is sweet.

Negation with quantifier

$\neg(\forall x \in \text{fruits}, P(x))$

$\exists x \in \text{fruits}$ s.t. $\neg P(x)$

$\neg(\exists x \in \text{fruits s.t. } (P(x) \wedge Q(x)))$

$\forall x \in \text{fruits}, \neg(P(x) \wedge Q(x))$

$\forall x \in \text{fruits}, \neg P(x) \vee \neg Q(x)$

	prove	disprove
$\forall$	* direct proof.	* counter-example
$\exists$	* example	* Same as prove universal.

* Proving a universal:

ex) For any integer k, if $\underbrace{\text{k is odd}}_{\text{hyp.}}$, then $\underbrace{k^2 \text{ odd}}_{\text{conclusion.}}$. Prove this statement

X DO NOT ASSUME CONCLUSION TRUE X

Let k be an integer, and odd. Then, $k = 2n+1$ where $n \in \mathbb{Z}$.

Then, $k^2 = (2n+1)^2 = \underline{4n^2+4n+1}$.

This can be written as $2(\underline{2n^2+2n})+1$.

Since $n \in \mathbb{Z}$, then $2n^2+2n \in \mathbb{Z}$. Let's call $2n^2+2n = j \in \mathbb{Z}$.

Then, $k^2 = 2j+1$, where $j \in \mathbb{Z}$, so by def'n of odd integers, k^2 is odd.

☆ Proving an existential.

ex) Prove that there exist integer k such that $k^2 = 0$.

$k = 0$ is such a $k \in \mathbb{Z}$;

$0^2 = 0$.

☆ Disproving a universal.

ex) Every rational q has a multiplicative inverse. Disprove this.

multiplicative inverse r of q is $r \in \mathbb{Q}$ such that $rq = 1$.

$q = 0$ does not. $r \cdot 0 = 0 \neq 1$.

☆ Disproving existential

There exists a $k \in \mathbb{Z}$ such that $k^2 + 2k + 1 < 0$. Disprove this.

For all $k \in \mathbb{Z}$, NOT $k^2 + 2k + 1 < 0$.

For all $k \in \mathbb{Z}$, $k^2 + 2k + 1 \geq 0$ ☆ prove universal.

Proof by cases

For all $j, k \in \mathbb{Z}$, if $\underbrace{\overbrace{\boxed{j \text{ is even}}}^{1} \text{ or } \overbrace{\boxed{k \text{ is even}}}^{2}}_{\text{hyp.}}$, then $\underbrace{jk \text{ is even}}_{\text{concl.}}$.

case 1: Assume $j, k \in \mathbb{Z}$, $\underline{j \text{ is even}}$. Then by def'n even, $j = 2n$, $n \in \mathbb{Z}$
Then, $jk = 2n \cdot k = 2(nk)$. $nk \in \mathbb{Z}$ since $n, k \in \mathbb{Z}$, so jk even.

case 2: Assume $j, k \in \mathbb{Z}$, $\underline{k \text{ is even}}$. Then ... $k = 2m$, $m \in \mathbb{Z}$.
Then $jk = j \cdot 2m = 2(jm)$. $j, m \in \mathbb{Z}$, so $jm \in \mathbb{Z}$. so jk even.